Geometrical and Vector Representation of Metrical Relations

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Abstract. *This paper enhances a model of derivative analyzes by adding the parameter of metric to its conceptual* corpus*. It is associated to the theoretical investigation related to the principle of developing variation in order to systematize the proper analytical identification of metric units in macro and micro scale, which result in the elaboration of new concepts. Their definitions, geometrical representations and practical analytical applications are shown, creating a new dimension in the formalization of the original model.*

Keywords*: Metric, Geometrical and Vector Representations, Analysis, Developing Variation.*

Resumo. *Este artigo aperfeiçoa um modelo de análise derivativa acrescentando o parâmetro de métrica a seu* corpus *conceitual. É associado a uma investigação teórica relacionada ao princípio de variação progressiva com objetivo de sistematizar a identificação analítica da unidade métrica em micro e macro escalas, o que resulta na elaboração de novos conceitos. Suas definições, representações gráficas e aplicações práticas em análises são apresentadas, criando uma nova dimensão na formalização do modelo original.*

Palavras-chave*: Métrica, Representações geométrica e vetorial, Variação progressiva.*

1. Introduction

This paper is part of an ongoing doctorate project which aims to comparatively analyze two contemporary violin sonatas, Op.78 by Johannes Brahms and Op.14 by Leopoldo Miguéz, considering the derivative relations employed in their thematic construction. The present study is associated to a theoretical investigation on the use of metric as a parameter for developing variation.[1] In order to systematize the proper analytical identification of metric unities in both large and small scales, new concepts were elaborated. Their

[1] Developing variation is the process "in which thematic materials are generated from the continuous modification of a very few motivic ideas" (Frisch 1993: 252). Arnold Schoenberg, who elaborated this concept, considered Brahms to be the most paradigmatic composer in the use of developing variation techniques.

definitions, as well as geometrical and vector representations are presented, followed by a practical analytical application.

One of the most distinctive characteristic of the model for the derivative analysis (MDA) is the abstraction of structural elements of a musical idea in separated streams, basically intervals and rhythms.[2] These are formally described in algebraic formulae, which are then considered in the derivative process. The present proposal intends to include metric as a new dimension in the formalization process. Besides that, the use of geometrical representation of metric structures aims to clarify its application.[3]

2. The *Zeitnetz*

This concept was originally elaborated by Justin London, in analogy to the widely well-known *Tonnetz*, intending to "map temporal relationships" (London 2001: 9). Influenced by principles derived from group and graph theories, London formatted his *Zeitnetz* (literally, a time net) as an infinite metric space which depicts numbers related to 2 and 3 ratios. Figure 1 shows a little section of the *Zeitnetz* (adapted from the original conception, see London 2001:10).

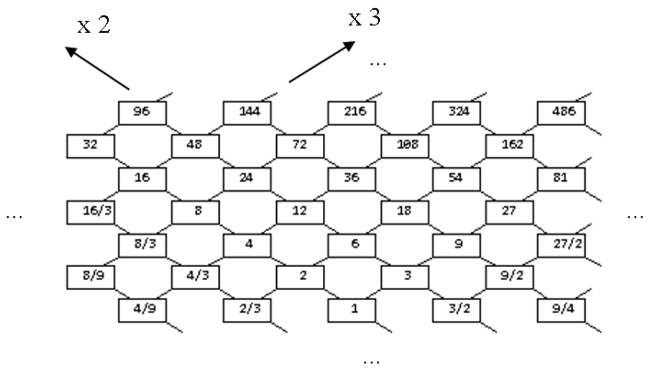

Figure 1: Partial representation of London's Zeitnetz. Numbers represent metrical proportions, and lines the associative ratios: 2 (southeast-northwest) and 3 (southwest-northeast). Plotted by the program PlotZeitnetz.

The *Zeitnetz* can be viewed as a hierarchical diagram of metrical relationships (by convention the unity is equivalent to a 16th note). Taking, for example, a quarter note (4) as the beat (or *tactus*) of a given musical passage, the next levels (sub-measure→ measure→ hypermeasure→…) will depend on the desired "ascending" trajectory, i.e., the "right" or "left" route. Figure 2 projects on the *Zeitnetz* some examples of metrical trajectories of conventional time signatures (for a better comparison, all of them limited to the range: sub-beat → beat(→sub-measure, if it is the case) → measure).

[2]For more detailed information about MDA, see XXX 2016.

[3]For a related approach considering geometrical representation of several musical structural aspects, see Timoczko 2011.

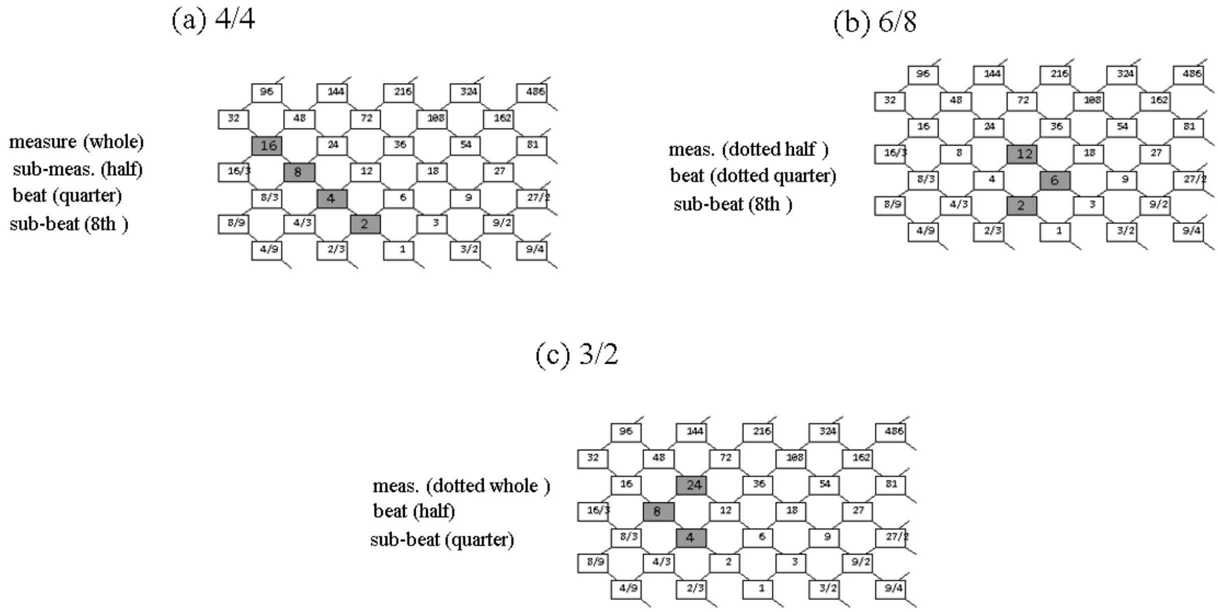

Figure 2: Representation on the Zeitnetz of three time signatures, considering embedded levels (indicated by the darker rectangles): (a) 4/4; (b) 6/8 and (c) 3/2. Plotted by the program PlotZeitnetz.

Intending for analytical and computational applications, the metrical trajectories on the *Zeitnetz* can also be represented by binary vectors (labeled mVs). With this purpose, the following conventions were stipulated: (i) the 2-route corresponds to the number "0"; (ii) the three-route corresponds to the number "1"; (iii) each vector's entry is associated to an upwards step on the *Zeitnetz* (i.e., in direction of a higher level). Thus, the three geometrical representations of Figure 2 can be translated as the following vectors: (a) <000>; (b) <10> and (c) <01>. Figure 3 associates these mVs to a more simplified graphic, in a tree-like format (they will be named "metrical trees" or mTs). Also, by convention, the first node of the tree will always correspond to the beat level (aiming at a better comparison, all of the three cases end at the measure level).

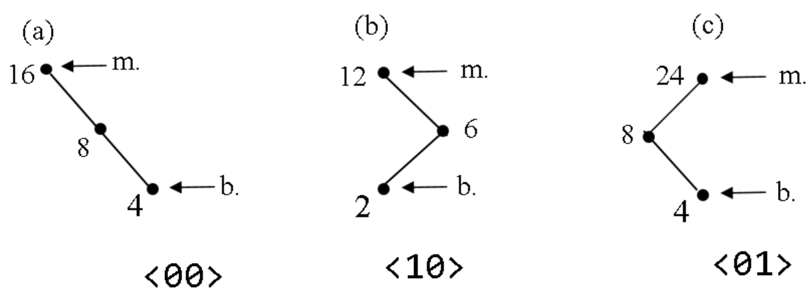

Figure 3: Concise representation of the metrical trajectories of Figure 2 as metrical trees. Numbers correspond to metrical proportions and arrows indicate beat level (b.) and measure level (m.). The respective mVs are shown below the trees.

3. Metrical profile and contour

Some specific needs of the research motivated the elaboration of a system destined to a precise quantification of the internal metrical configuration of brief motivic ideas. In opposition to the global (or macro) scope of *Zeitnetz* (intended to map relatively larger musical extensions), this approach addresses local (or micro) analysis, in general, encompassing one or two bars-long passages.

According to the conventions adopted by the derivative analytical model, any contextualized musical segment (a motive, for example) can be considered as a multidimensional set of structural information. These can be abstracted and mathematically described, which corresponds to the motive's formal identification. Four domains were idealized as subjected to abstraction: (1) sequence of melodic intervals; (2) intervallic range; (3) sequence of rhythmic durations and (4) metric configuration. Until now only the three first cases were satisfactorily described in the model. This section proposes a new approach for the metrical parameter, firstly by formalizing it as a matrix, and then extracting from this a geometric representation.

The construction of the metrical matrix (mM) of a melodic segment "X" obeys the following guidelines: (i) determine the metric context of X (in the other words, its time signature); (ii) determine the number of rhythmic levels present in X (it is obtained by the difference between the longest and the shortest durational figure); (iii) attribute numeric values to each onset of X, according to its relative metric position, "0" being for "strong" and "1" for "weak", considering each rhythmic level of X. Figure 3 exemplifies the procedure with a simple rhythmic abstract motive. Its durational sequence is numerically expressed as <4 2 2 6 8>, considering the 16th note as the unity. There are three metrical levels in this example, which are depicted below the motive. Intuitively, one can consider that the more deeply rooted is a given event, the higher must be its metric significance (for instance, in this aspect event 1 is clearly stronger than event 2, which is stronger than event 3).

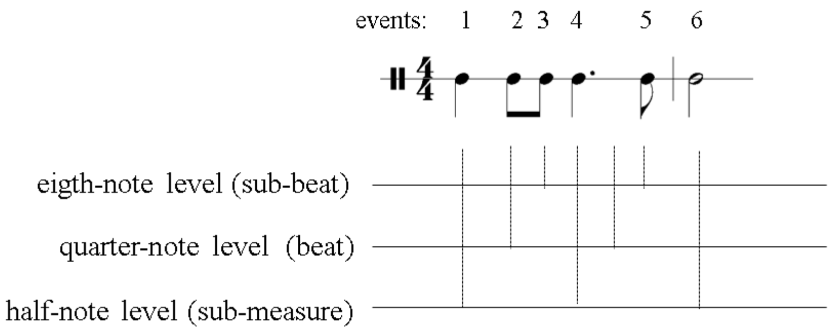

Figure 3: Example of a rhythmic motive with its metrical levels.

Table 1 presents the metrical matrix associated to Figure 3. As we can observe, events 1 and 6 are the metrically strongest, being notated as “000”, while event 5 is the weakest (111).

Table 1: Metrical matrix of motive X (c.f. Figure 2).[4]

event	levels		
	1	2	3
1	**0**	**0**	**0**
2	**0**	**1**	**0**
3	**0**	**1**	**1**
4	**1**	**0**	**0**
5	**1**	**1**	**1**
6	**0**	**0**	**0**

The content of the mM of a given motive X can be quantified, resulting in its *metrical profile* (mP). Basically, it is done by attributing to each event a sort of score that will represent its relative importance inside X's local metrical context.

The score of each event present in mM is calculated according to the formula:

$$s_e = \prod_{e=1}^{n} \left[1 - \left(be * \frac{Le}{10}\right)\right]$$

where s_e is the metric score of the event *e*, *n* is the number of levels, *be* is its corresponding binary digit and *Le* its corresponding level number.

The application of this formula to the exemplified mM result in the following scores (Table 2):

Table 2: Calculation of metric scores corresponding to the mM of motive X(c.f. Figure 2).[5]

event	levels			metric score
	1	2	3	
1	0	0	0	{[1- (0*1/10)]*[1-(0*2/10)]*[1-(0*3/10)]} = 1.00
2	0	1	0	{[1- (0*1/10)]*[1-(1*2/10)]*[1-(0*3/10)]} = 0.80
3	0	1	1	{[1- (0*1/10)]*[1-(1*2/10)]*[1-(1*3/10)]} = 0.56
4	1	0	0	{[1- (1*1/10)]*[1-(0*2/10)]*[1-(0*3/10)]} = 0.90
5	1	1	1	{[1- (1*1/10)]*[1-(1*2/10)]*[1-(1*3/10)]} = 0.50
6	0	0	0	{[1- (0*1/10)]*[1-(0*2/10)]*[1-(0*3/10)]} = 1.00

The sequence of the computed scores (<1 0.80 0.56 0.90 0.50 1>) forms the *metrical profile*(mP) of the motive, which can be plotted as a bar graph (Figure 4).

[4] The computation of a mM is automatically done by the program *MetricProfile*.
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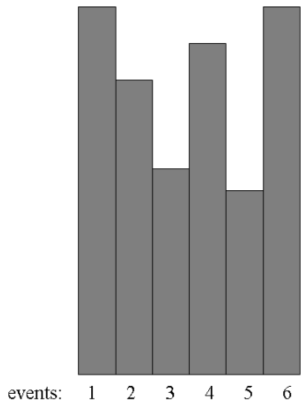

Figure 4: Representation ofthe metric profile of motive X(Figure 2).

However, aiming at comparing mPs of different motivic ideas in an eventual analysis (a central issue of the research), it seems more relevant to generalize the graphs rather than to maintain their absolute values. For this, a mP is translated as a *metric contour* (mC), a graph which, as well as a mP, informs the internal correlations between the metrical fluctuations of the rhythmic events of a given motive, but in a *relative* rather than an absolute, configuration (Figure 5).

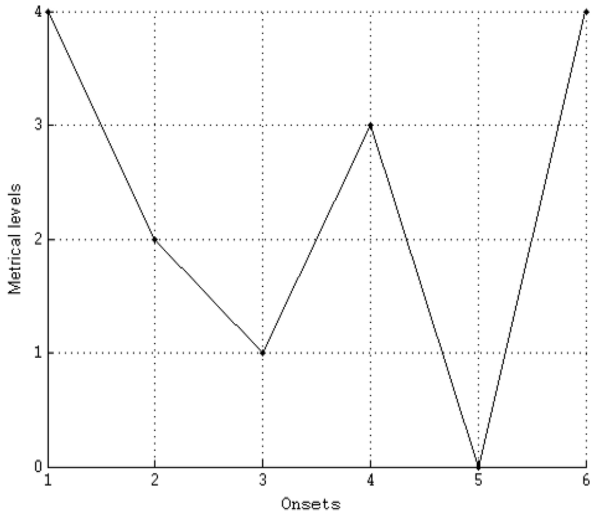

Figure 5: Representation ofthe metric contour of motive X(Figures 2 and 4). Axe x depicts the onsets (events), axe y presents the relative metric levels. Plotted by the program MetricProfile.

The contour confirms, as it was shown by the vectors, that events 1 and 6 are the metrically strongest. Event 5 is the weakest and events 2, 3 and 4 occupy intermediary positions. The sequential numeric description of the contour (i.e., 4-2-1-3-0-4) constitutes the definitive metric profile of the motive, being notated as a vector: <421304>.

4. A practical application

Brahms' Violin Sonata Op.78 was selected for providing examples of the potential applications of the concepts here introduced. At first, aiming at local metrical dimension,

Figure 7 presents the sonata's *Grundgestalt*,[6] encompassing four motivic components, labeled as A to D.

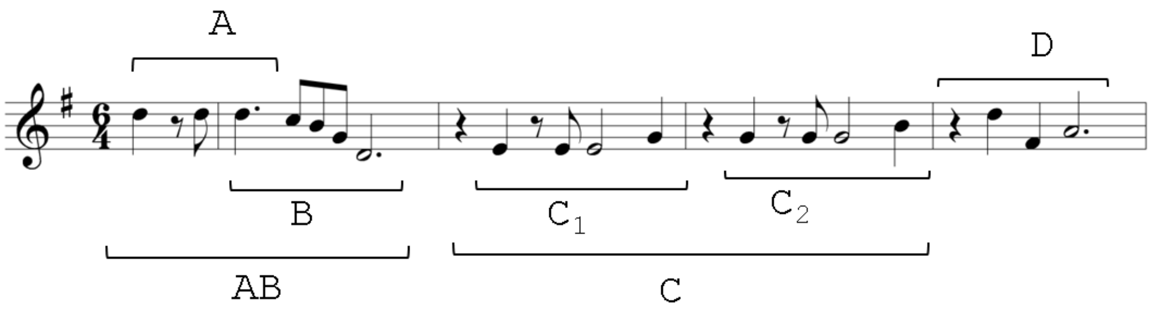

Figure 7: Brahms – Violin Sonata Op.78 (mm.1-4) – Grundgestalt subdivided into components A-D.

According to MDA conventions, the analysis of a *Grundgestalt* is initiated with the abstraction of an intervallic and a rhythmic sequence from each component. Recently, the intervallic range was also included among the abstracted structures. Figure 8 exemplifies this initial process, selecting for exam the component B.

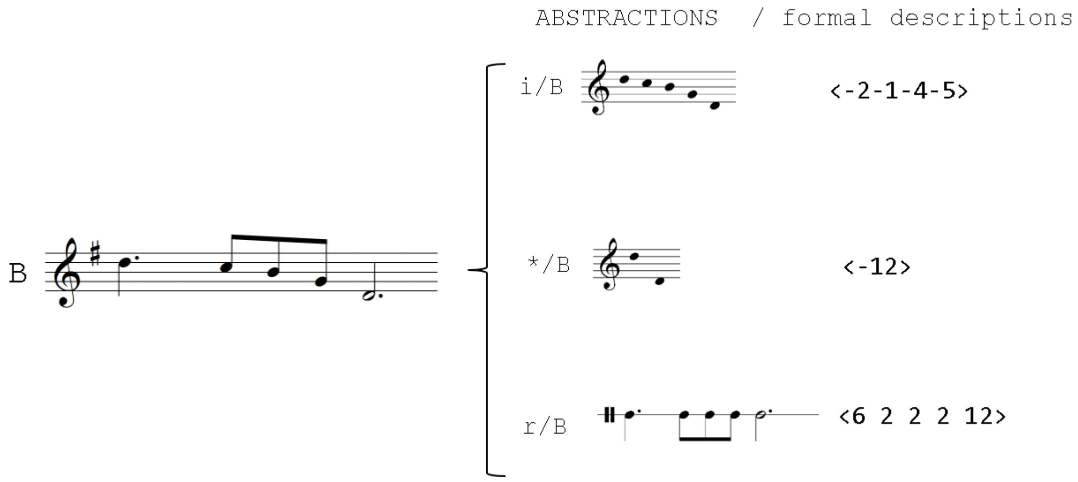

Figure 8: Grundgestalt's component B and its abstractions: intervallic sequence (i/B) and range (*/B), rhythmic sequence (r/B). Their formal, vector descriptions are added at right.

The concepts presented in this study allow the addition a new dimension – namely, the metric configuration – to the current group of abstractions. With the time signature (6/4) of the piece (that is, B's local metrical context) and the content of the rhythmic sequence r/B (<6 2 2 2 12>) we obtain B's mM and the metric scores of its five events (Table 3).

[6] In a very concise definition, a *Grundgestalt* (normally translated as basic shape) corresponds to a group of brief musical ideas (generally presented at the beginning of a piece) from which a lot of thematic and motivic material can be extracted through extensive processes of variation. As developing variation, this concept was elaborated by Schoenberg and is originated from an organicist conception of musical creation.

Table 3: Metrical matrix of the five events present in B and their respective metric scores (c.f. Figure 8).

event	levels			metric scores
	1	2	3	
1	**0**	**0**	**0**	1.00
2	**0**	**1**	**1**	0.56
3	**0**	**1**	**0**	0.80
4	**0**	**1**	**1**	0.56
5	**1**	**0**	**0**	1.00

Figure 9 plots the metrical contour of B, finally producing its metrical profile. In essence, it can be considered the formal vector description of B's metrical abstracted component (or, in the model terms, m/B).

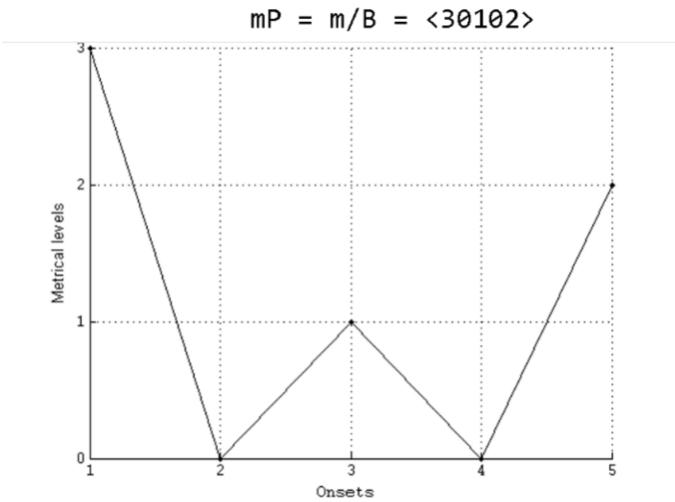

Figure 9: Metrical contour and metrical profile of B.

In a global approach, we can use the metrical trees to evidence metrical conflicts.[7] Figure 10 selects two instances of superposition of distinct metrical configurations between violin and piano in the very beginning of the first movement of Brahms' Op.78.

[7]The use of metric as a structural means for producing developing variation is one of the most distinctive traits of Brahms compositional procedures (see, among others, Smith 2007, Ng 2005 and Frisch 1984).

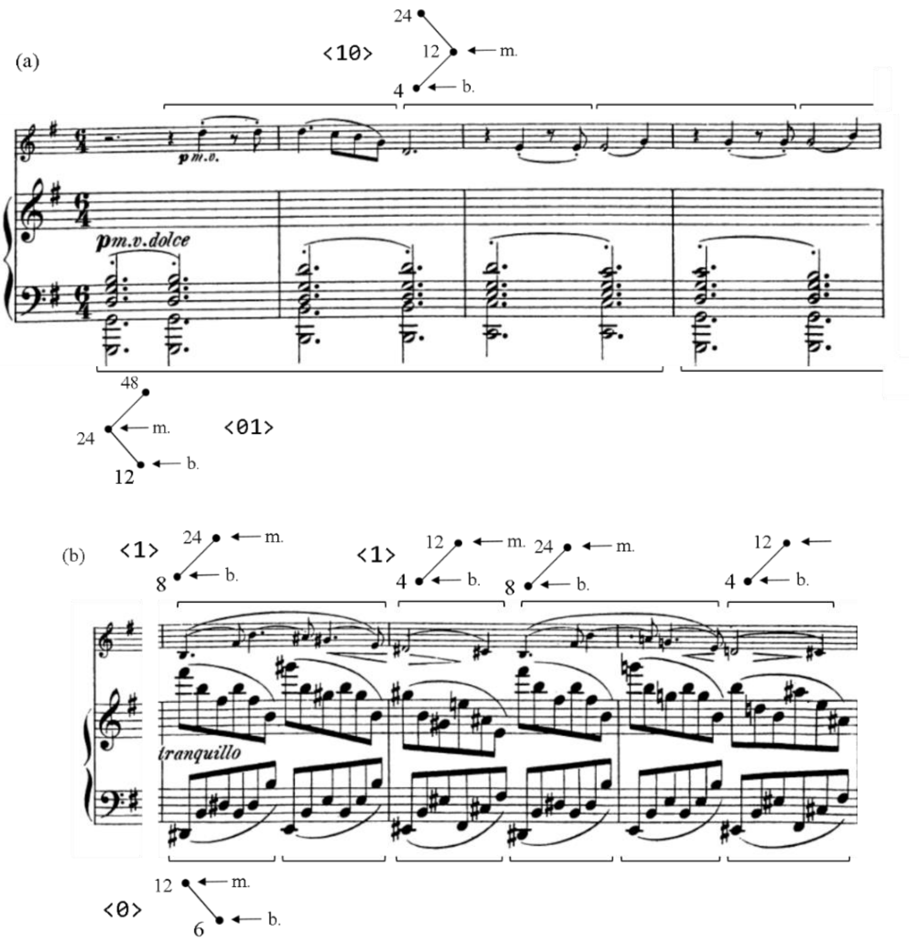

Figure 10: Brahms – Violin Sonata Op.78/I (mm.1-4) – two instances of metrical conflict between violin and piano: (a) mm.1-4; (b) mm.11-13.

In case (a) the conflict results from the different metrical phases of theme (violin) and accompaniment (piano). While the low-register chords clearly depict the noted binary metric (suggesting a more subtle hypermetric configuration encompassing three measures, due to the return of the tonic harmony in m.4), the thematic line seems to be in 3/4 time signature and, moreover, presenting its strong beats occurring dislocated in relation to the respective points of the piano part (note that this causes difference of phases in the hypermetric level).[8] The metrical trees and, specially, the mirrored vectors capture these incompatibilities between both planes. The passage in Fig.10b constitutes a more "classical" case of hemiola. As pointed out by Walter Frisch (1984: 78), the piano configuration is implicitly structured as a sequence of binary 6/8 measures, over which the violin seems to alternate two ternary, not-notated, time signatures: 3/2 and 3/4.

It is also possible to express the two situations using an alternative, more concise kind of graphic representation (Figure 11). Even if there is clearly hemiolic construction in both cases,[9] they have subtle, different natures: while the latter presents a regular, recurrent

[8]As suggested in Figure 10(a), the first strong beat of the violin line occurs at the second half of notated measure 2.

[9]The antagonism between binary and ternary meters is clearly reflected by the opposition 0/1 of the superimposed vectors.

pattern, the former violin and piano groups interact in a mutually independent way, in a manner similar to tectonic plates.

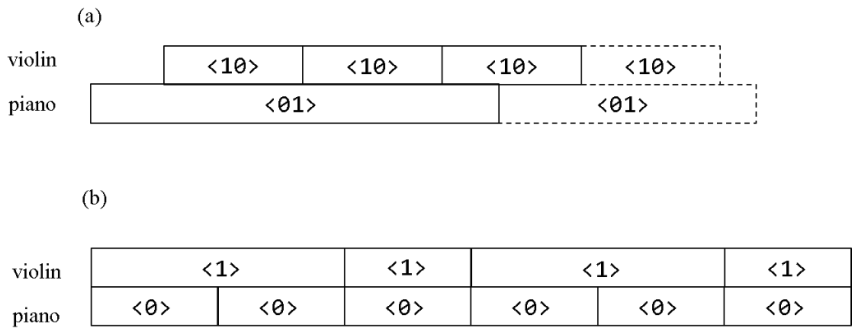

Figure 11: Alternative representations of metrical conflicts of Figure 10. The dotted-line rectangles in (a) indicate possible continuations.

5. Concluding remarks

We believe that the new concepts introduced in this study (mV, mT, mM, and mP/mC) constitute a meaningful contribution for the expansion of metric theory. Vector and geometrical representations of these concepts function as complementary resources which allow not only their understanding but also systematization. As it was demonstrated in the previous section, their integrated application in analytical approaches provides robust means for examining the metric dimension in both large and small scales, evidencing structural details and relationships that otherwise possibly would not be available. In this sense, it is relevant to mention that some computational tools were developed for assisted analysis.

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